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RECRUITER PRODUCTIVITY
AND THE
POISSON DISTRIBUTION

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Military recruiting productivity may be viewed as the number of successful enlistments in a given recruiting time interval, and also as a Poisson-distributed random variable. One measure of effectiveness for recruiters is the probability of meeting a specified minimum number of enlistments (making mission). The Poisson model permits investigation of the impact on this performance measure when the length of the mission period is changed, to when recruiter production is aggregated as in station missioning. Immediate results are that less effective recruiters benefit from shorter mission periods, while the effective recruiter will benefit from longer mission periods. Also, estimates of this probability should be improved with the Poisson model, rather than treating the attribute of making mission as a Bernoulli trial.

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SUMMARY

The number of successful enlistments in a given recruiting interval is considered as a Poisson-distributed random variable, and properties of the Poisson are used to examine various aspects of the recruiting process. An immediate result is that with a mission of two enlistments per month, a recruiter whose average performance is the mission level of two per month will have only a 60% chance of making mission. To have a 90% chance of making mission, a recruiter must perform, on the average, at least 40% better than the mission value.

The chance of making mission is sensitive to the length of the mission period, even when seeming adjustments are made in the size of the mission. Doubling both the mission and the time period to achieve the mission will, for the same recruiter, change the probability of making mission. It appears that when the probability of making mission is to be computed as the MOE, less effective recruiters will benefit from shorter mission periods, while the effective recruiter will benefit from longer mission periods. Similar results occur when the performance of several recruiters is combined, as would be done in station missioning. It is clear that direct comparisons between different time periods or among several individual recruiters on stations should always involve mission periods of the same length.

When the measure of effectiveness is the probability of making mission, estimation of that probability should be improved when the number of recruiting successes is treated as Poisson distributed. In general, the size of resulting confidence intervals seems smaller when recruiting results are treated as Poisson than when, for a recruiting period, the attribute of making mission is treated as a Bernoulli trial.

This paper is exploratory in that detailed analyses of these results were not undertaken. Rather, we have looked at the kinds of contributions the Poisson model can make to our understanding of the recruiting process.

RECRUITER PRODUCTIVITY
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A useful premise in studying recruiter productivity is that the number of recruiting successes in a specified time interval is a *random variable*. This means that for a measurement time period of one month, for example, the number of recruiting successes will vary from month to month even if other factors remain constant. These differences are due to random variations rather than real changes in performance. Similarly, a recruiter or team of recruiters operating with the same performance level will produce different numbers of recruiting successes from month to month. If x is the number of successes in a time period, then we shall portray x as a discrete random variable with possible values $x = 0,1,2, \ldots$, and we wish to be able to compute the probabilities of each of these values occurring.

In the following sections we shall examine one way of computing such probabilities. First, we shall suggest the Poisson distribution as a means of finding the probabilities of various values of recruiting successes, and of computing the chances of making mission. Then we shall describe useful properties of the Poisson in the framework of a recruiting scenario, showing how it may be used with multiple time periods of varying length, and with combined performance of several recruiters who operate at different performance levels. We shall apply this representation to several important concerns in the evaluation of recruiting results.

The Poisson Distribution

A desirable property for the distribution *form* for x, the number of recruiting successes in a time period, is that this form should not vary because of the length of the time period. This means that if x had distribution form $p(x)$ for each month with average or mean value $E(x)$, then the distribution form for three months of production $y = x_1 + x_2 + x_3$ should be functionally the same with an average for the three-month period of $E(y) = 3E(x)$. The Poisson distribution has this property.

The number of recruiting successes in a time period is to be considered as a discrete, integer-valued random variable bounded below by zero, so that $x = 0,1,2, \ldots$. The Poisson distribution represents such a random variable.

Finally, and most important, there should be some real-world justification for choosing the Poisson. The Poisson has been shown to arise in many natural settings where the random variable is the number of events occurring in a given time period (Ref 1). Applications vary widely, and include the number of goals scored by a soccer player, immigrant counts, pulses on a Geiger counter, bacteria counts, flying bomb hits in London (WWII), and so on. In fact, one of the first observations of the Poisson arising in nature was in a military setting, in that it properly represented the number of Prussian soldiers killed by kicks from horses (Ref 2). Two widely studied applications of the Poisson are in the study of waiting lines, and in Reliability theory. In waiting lines work was begun by A.K. Erlang at the Danish Telephone Company in 1909, with the number of arriving calls being the Poisson-distributed random variable (Ref 3). In the study of system and component reliabilities, the use of the Poisson is to represent the number of failures per unit time period, where the Poisson parameter is called the *failure rate*, and its reciprocal is the MTBF, or *mean time between failures* (Ref 4). In these examples the time between event arrivals is considered memoryless, and the number of event arrivals per unit time, Poisson.

The mathematical form for the Poisson distribution is

$$P(x) = \frac{e^{-\lambda}\lambda^x}{x!}, \quad x = 0,1,2,\dots .$$

In application computations using this functional form are rarely undertaken, with reliance on standard tables of the cumulative form or computer packages. From this point forward in this paper we shall simply let $P(x)$ be the probability of x recruiting successes occurring in the specified time interval, and use standard tables for numerical examples.

Properties of the Poisson Distribution

In its pure mathematical form the Poisson distribution has one parameter λ, and a random variable following this distribution will have a mean

$$E(x) = \lambda,$$

and a variance (Ref 1)

$$\mathrm{Var}(x) = \lambda.$$

Thus a useful and simplifying attribute of Poisson-distributed random variables is that they have the same numeric value for their mean and their variance.

In applications where x represents the number of arrivals in a time interval of length t, the parameter λ is represented as the product of t and the arrival rate. In applying this distribution to recruiter productivity we shall take this parameter as the product of the *recruitment rate r*, and the *recruitment interval t* , and say that the number x of recruiting successes in the time interval of length t at recruitment rate r is Poisson distributed with parameter

$$\lambda = rt.$$

If we measure time in units of one month, the recruitment rate should be measured in successes per month. Note that neither rate r nor time t need be integer valued. The time period could be, for example, $t = 2.3$ months.

Here is an example of a set of Poisson probabilities. If we are interested in the number of recruiting successes in one-month time periods, and the recruitment rate r is two successes per month, the average per month is $rt = 2.0$ (which is also the Poisson parameter) and the Poisson probabilities are shown in Table 1 below.

Table 1. Poisson Probabilities and Cumulative Probabilities when the Mean Number is 2.0 per Month

x	Probability of x	Probability of x or less
0	0.135	0.135
1	0.271	0.406
2	0.271	0.677
3	0.180	0.857
4	0.090	0.947
5	0.036	0.983
6	0.012	0.995
7	0.004	0.999
8	0.001	1.000

This example shows that although the Poisson-distributed random variable lacks a distinct upper bound (as do recruiting successes per month), the probabilities become negligible as the random variable reaches high values beyond the mode of its distribution. It may also be seen for this example that if the mission were four per month, the probability of not making mission is the probability of three or less, or 0.857. Similarly, the chance of making a mission of four is the chance of four or more, or $1 - 0.857 = 0.143$.

A general Poisson property is that the sum of independent poisson-distributed random variables is itself Poisson. Thus if x_1 is Poisson with parameter (and mean) r_1t_1, and independent random variable x_2 is Poisson with parameter (and mean) r_2t_2, then the sum $x_1 + x_2$ will be Poisson distributed with parameter (and mean) $r_1t_1 + r_2t_2$. For a recruiting scenario, for example, the combined output of two independently operating recruiters performing at r_1 and r_2 successes per month, respectively, will be Poisson with mean and parameter $r_1t + r_2t = (r_1 + r_2)t$. Also, a single recruiter operating at rate r successes per month will produce a total for three months which is Poisson with mean $3r$. We'll look at several recruiting implications of these Poisson properties in the next section.

How Many Months for a Mission?

Does the length of the mission period influence the chance of making mission? Can the chance of a recruiter being successful be changed by simply changing the time period over which the mission is defined?

We'll present an example where the starting place is a mission of $M = 2$ for a one-month period. Then, we'll contrast a recruiter's chance of making mission for a one-month period, with two seemingly identical missions:

a. A mission of $M = 1$ for a 0.5 month period, and

b. A mission of $M = 4$ for a two-month period.

For our example, let the recruiter operate at the "mission level". This means that the expected productivity is at the mission level, or that the mean of the Poisson distribution for this recruiter is $rt = M$. For a mission of $M = 2$ in a one-month period, such a recruiter would be characterized by recruitment rate of $r = 2$ in that the recruiter would produce, on the average, two successes per month. The chance of making mission is *

$$\begin{aligned}\Pr(\text{Make Mission}) &= \Pr(x \geq 2)\\ &= 1 - \Pr(x \leq 1)\\ &= 1 - 0.406\\ &= \underline{0.594}.\end{aligned}$$

If we reduce the recruiting interval to 0.5 months, the mission should be halved to $M = 1$, and the recruiter (now at $rt = 2(0.5) = 1.0$) has a chance of making mission of

$$\begin{aligned}\Pr(\text{Make Mission}) &= \Pr(x \geq 1)\\ &= \underline{0.632}.\end{aligned}$$

---

* Here we use the $\Pr(x \leq 1)$ value from the example of Poisson probabilities shown in Table 1.

On the other hand, doubling the recruiting interval to two months (and the mission to $M = 4$) gives $rt = 2(2) = 4.0$, and the recruiter's chance of making mission is now

$$\Pr(\text{Make Mission}) = \Pr(x \geq 4)$$
$$= \underline{0.567}.$$

Clearly, the three probabilities for the example above are not substantially different from one another.

Similar calculations may be made for a recruiter who performs at a less-than-mission level (say, $r = 1$), and for a recruiter who performs at a more-than-mission level (say, $r = 3$). Results for all three recruiters are summarized in Table 2.

Table 2. Mission Success Probabilities With Various Recruiting Intervals

	Probability of Making Mission		
Interval Mission Success	0.5 Months 1 $x \geq 1$	1 Month 2 $x \geq 2$	2 Months 4 $x \geq 4$
A less-than Mission-Level Recruiter, $r = 1.0$	0.393	0.264	0.143
A Mission-Level Recruiter, $r = 2.0$	0.632	0.594	0.567
A more-than Mission-Level Recruiter, $r = 3.0$	0.777	0.801	0.849

The example in Table 2 shows that a recruiter's chance of making mission *can be changed* by changing the time interval over which the mission is defined, even while adjusting the mission value. These data suggest that a less effective recruiter will benefit from a shorter recruiting interval, whereas an effective recruiter will benefit from longer mission periods.

The influence of mission period on the chance of making mission is not necessarily surprising. Intuition suggests that longer intervals will allow the effective recruiter to make better use of his surpluses, while shorter periods permit the ineffective recruiter more chances at curtailing the size of his shortage. What does seem clear, however, is that recruiting performance comparisons using the probability of making mission should always involve the same size of recruiting interval on which the mission is defined.

The Recruiter Who Makes Mission 90% of the Time

As we have seen for a mission of size $M = 2$, a mission-level recruiter (one whose average number of recruiting successes is equal to the mission value) will only make mission about 60% of the time. How productive must a recruiter be to make mission 90% of the time?

For a recruiting interval of one month, the chance of making mission for recruiters who are mission-level, 50% better, and twice mission level, are shown in Table 3.

Table 3. Mission Success Probabilities for Various Recruiters and a One-Month Recruiting Interval

	Probability of Making Mission		
Mission M	1	2	3
A Mission-Level Recruiter, $r = M$	0.632	0.594	0.577
A Recruiter who is 50% better than Mission Level, $r = (1.5)M$	0.777	0.801	0.826
A Recruiter who is twice Mission Level, $r = (2)M$	0.865	0.908	0.938

In this example three possible mission values are used, and for these cases we see that even being a recruiter who is 50% better than mission level doesn't provide a 90% chance of making mission. For a mission of $M = 2$ in one month, one has to be twice as good as a mission level recruiter to attain that 0.90 probability.

Table 4 below generalizes these results. Recruiter performance r is taken as a multiple of mission value, so that $r = yM$, and Table 4 shows minimal values of y needed to attain, for various missions, a 90% chance of making mission. These values hint at asymptotic behavior at around 40% better than mission level, for one-month missions for individual recruiters.

Table 4. Recruiter Performance Levels for a 90% Chance of Making Mission

Mission M	Recruiter performance $r = yM$	Multiple y
1	2.4	2.4
2	4.0	2.0
3	5.4	1.8
4	6.8	1.7
5	8.0	1.6
6	9.5	1.6
7	11.0	1.6
8	12.0	1.6
9	13.0	1.4
10	14.5	1.4
15	22.0	1.4

Combining Recruitment Results: Two Recruiters

The fact that the sum of Poisson-distributed random variables is itself Poisson gives us the chance to look at results when enlistments from several recruiters are combined. Results here mirror those given earlier when we looked at the length of the mission period, and examples are shown in Table 5. It is important to emphasize that these results assume that the Poisson results which are combined are *independent.* What is shown are results for recruiters who are continuing to operate independently, and not as teams. The combining is done in scorekeeping.

Table 5. Mission Success Probabilities When Recruiters are Grouped

Recruiter Pairing	Individual Mission	Individual Chance	Combined Mission	Combined Chance
Two Mission-Level Recruiters	2 2	0.594 0.594	4	0.567
One Mission-Level and one 50% better-than Mission	2 2	0.594 0.801	4	0.735
Two 50% Better-than Mission Level	2 2	0.801 0.801	4	0.849
Two Twice Mission-Level Recruiters	2 2	0.908 0.908	4	0.958

Table 5 shows that, as was the case when the time period was increased, the result of combining results from good recruiters is a greater chance of making mission, whereas the chance is reduced when results from ineffective recruiters are combined. Here, as in previous examples, we have only looked at some trial values and not done a complete analysis to determine where the switch occurs.

Combining Recruitment Results: Station Missioning

Results such as those shown in Table 5 can be expected when enlistments from more than two recruiters are combined, as would be the case in *station missioning*. The chance of the station making mission will exceed that of individuals who are good performers.

Bernoulli Trials and the Poisson Distribution

One measure of effectiveness for a recruiter's performance is, simply, the probability of making mission. Here the (1,0) values for the Bernoulli variable are assigned to the events of making mission, or not making mission, and the outcome of the mission period is considered a Bernoulli trial. In this way making mission is modelled in a simple attribute or yes-no manner, without direct reference to the actual number of recruiting successes.

The "counting data" generated from multiple Bernoulli trials is often described by two probability distributions. For a value of parameter p, the geometric distribution gives probabilities for the number of consecutive Bernoulli trials until the first success. The binomial distribution gives the probabilities for the number of success in a set of n independent Bernoulli trials, each with parameter p. Here, for example, a frequent application is to estimate p on the basis of data showing that in n periods, the recruiter made mission x times.

Inferential statistics for our MOE p based on such counting data suffer because of the simple attribute measurement as Bernoulli trials. Large samples are needed to avoid large confidence intervals for estimation of p, or soft OC curves for hypothesis tests for p. Data from 50 or 100 mission periods may be needed to produce a useful estimate of p. If the recruiter makes mission x times in n periods, the point estimate for his measure of effectiveness is $p = x/n$, and a 95% confidence interval for a mid-range p value can be as large as

$$p \pm 1.96\ (0.5)\ (1\text{-}0.5)/n,$$

and while this is a "worst case" example, it is readily seen that even for $n = 100$ recruitment intervals, the 95% confidence interval for p will be $\pm\, 0.10$. If a recruiter makes mission twelve times in twenty periods, confidence bounds from the binomial distribution are 0.37 and 0.82, results which clearly have little utility.

Describing events as Bernoulli trials is necessary where all that is observed is the presence of the attribute: the event happened, or it did not. In making mission this is not the case because we are using a finer measure, viz., the actual number of recruiting successes achieved in the period. We compare this variable measure with mission M to determine if the attribute (made mission) is present or not. For a given number of periods n, confidence intervals for estimating p should be smaller if based upon the variables measure (number of successes, Poisson distributed) rather than the attribute measure (made mission or not).

Examples of Estimating the Probability of Making Mission

Continuing our exploration of the relationship to Bernoulli trials, here is an example of the estimation of p, the probability of making mission. For the same data set we will generate a 95% confidence interval for p treating the data as 18 Bernoulli trials (attributes), and then find the confidence interval of p using the variables data and the Poisson distribution.

Let the recruiting interval be one month, and the mission be $M = 2$ for a one-month period. Generated using random numbers, eighteen months of recruiting successes per month are:

1	2	0
1	2	1
1	0	3
2	1	2
2	1	2
0	1	3

.

Treating these as Bernoulli trials, we see that the mission of $M = 2$ was achieved eight times, and the point estimate of the probability of making mission is 8/18 = 0.444. Of course, this estimate does not reflect the number of months. Viewing the data as eighteen independent Bernoulli trials, each with parameter p, gives a 95% confidence interval for p of

$$0.22 \leq p \leq 0.68$$

when standard curves from the binomial distribution are employed (Ref 5). An alternate way to find this confidence interval uses the normal approximation to the binomial, and gives the similar result:

$$0.21 \leq p \leq 0.67.$$

When the data is viewed as variables data from a Poisson distribution, confidence limits for the probability of making mission may be obtained by the following procedure. The data has a sample mean of 1.389 and sample standard deviation of 0.916. The sample size of eighteen allows us to invoke the central limit theorem regarding normality of the sample mean. Authorities agree that 95% confidence limits for the Poisson mean are (Ref 1)

$$\bar{x} \pm 1.96\sqrt{\frac{\bar{x}}{n}},$$

which with our data yield

$$0.845 \leq rt = \lambda \leq 1.933.$$

Since the probability distribution function is monotone nondecreasing with the population mean, these bounds can be used to obtain bounds for the chance that the number of successes is one or fewer:

$$0.424 \leq 1 - p \leq 0.792.$$

Finally 95% confidence limits for the probability p of making mission are

$$0.208 \leq p \leq 0.576.$$

Confidence intervals for p obtained in this manner should, in general, be smaller than those obtained using the Bernoulli trial, binomial approach.

Discussion

On the previous pages we have looked briefly at some of the results that may be obtained when the number of successful recruitments in the mission period is treated as a Poisson-distributed random variable. Individual recruiter productivity was characterized by the parameter of the Poisson, and we have shown some relationships between this parameter and the probability of making mission.

In applying this representation to several recruiting questions, we have used numeric examples rather than thorough analyses. For example, we have shown that the length of the mission period influences effective and ineffective recruiters differently, but have not yet explored where, in mission size and length, the change occurs. Similarly, our numeric example gave a smaller confidence interval from the Poisson than from the binomial distribution. However, in our example neither confidence interval was small enough to be useful in estimation. Not yet addressed was the more general question of how many observations (mission periods of data) to estimate the probability of making mission are *needed* with each approach. Another topic we have only addressed briefly is that of station missioning. The Poisson distribution will permit detailed analysis of the impact of combining recruiter results through station missioning. Such a study could also look into results when teamwork improves performance.

How appropriate is the Poisson distribution? Perhaps the most serious shortcoming in the work presented here is not so much the use of the Poisson to represent counts of randomly arriving successful recruitments, but rather, that we assumed that the recruitment rate (or Poisson parameter) would be constant throughout the mission period. There appear to be data suggesting that, if mission has not been made, the recruitment rate will increase toward the end of the mission period. Here the distribution for shorter time periods could still be Poisson, but with an increasing parameter. Arrivals early and late in the mission period would not necessarily be independent events.

The Poisson distribution appears to offer a fertile means of investigating some aspects of the recruiting process. It is hoped that the suggestions in this paper will lay the path to more complete studies.

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